RELATIVISTIC ELECTRON BEAMS

Here $\gamma = (1-\beta^2)^{-1/2}$ is the relativistic scaling factor; quantities in analytic formulas are expressed in SI or cgs units, as indicated; in numerical formulas, I is in amperes (A), B is in gauss (G), electron linear density N is in cm^{-1}, and temperature, voltage and energy are in MeV; $\beta_z = v_z/c$; k is Boltzmann's constant.

Relativistic electron gyroradius:

$$r_e = \frac{mc^2}{eB}(\gamma^2-1)^{1/2} \text{ (cgs)} = 1.70 \times 10^3 (\gamma^2-1)^{1/2} B^{-1} \text{ cm}.$$

Relativistic electron energy:

$$W = mc^2\gamma = 0.511\gamma \text{ MeV}.$$

Bennett pinch condition:

$$I^2 = 2Nk(T_e + T_i)c^2 \text{ (cgs)} = 3.20 \times 10^{-4} N(T_e + T_i) \text{ A}^2.$$

Alfvén-Lawson limit:

$$I_A = (mc^3/e)\beta_z\gamma \text{ (cgs)} = (4\pi mc/\mu_0 e)\beta_z\gamma \text{ (SI)} = 1.70 \times 10^4 \beta_z\gamma \text{ A}.$$

The ratio of net current to I_A is

$$\frac{I}{I_A} = \frac{\nu}{\gamma}.$$

Here $\nu = Nr_e$ is the Budker number, where $r_e = e^2/mc^2 = 2.82 \times 10^{-13}$ cm is the classical electron radius. Beam electron number density is

$$n_b = 2.08 \times 10^8 J\beta^{-1} \text{ cm}^{-3},$$

where J is the current density in $\mathrm{A\,cm}^{-2}$. For a uniform beam of radius a (in cm),

$$n_b = 6.63 \times 10^7 I a^{-2}\beta^{-1} \text{ cm}^{-3},$$

and

$$\frac{2r_e}{a} = \frac{\nu}{\gamma}.$$

Child's law: (non-relativistic) space-charge-limited current density between parallel plates with voltage drop V (in MV) and separation d (in cm) is

$$J = 2.34 \times 10^3 V^{3/2} d^{-2} \text{ A cm}^{-2}.$$

The saturated parapotential current (magnetically self-limited flow along equipotentials in pinched diodes and transmission lines) is[29]

$$I_p = 8.5 \times 10^3 G\gamma \ln\left[\gamma + (\gamma^2 - 1)^{1/2}\right] \text{ A},$$

where G is a geometrical factor depending on the diode structure:

$G = \dfrac{w}{2\pi d}$	for parallel plane cathode and anode of width w, separation d;
$G = \left(\ln \dfrac{R_2}{R_1}\right)^{-1}$	for cylinders of radii R_1 (inner) and R_2 (outer);
$G = \dfrac{R_c}{d_0}$	for conical cathode of radius R_c, maximum separation d_0 (at $r = R_c$) from plane anode.

For $\beta \to 0\ (\gamma \to 1)$, both I_A and I_p vanish.

The condition for a longitudinal magnetic field B_z to suppress filamentation in a beam of current density J (in A cm^{-2}) is

$$B_z > 47\beta_z(\gamma J)^{1/2} \text{ G}.$$

Voltage registered by Rogowski coil of minor cross-sectional area A, n turns, major radius a, inductance L, external resistance R and capacitance C (all in SI):

externally integrated	$V = (1/RC)(nA\mu_0 I/2\pi a)$;
self-integrating	$V = (R/L)(nA\mu_0 I/2\pi a) = RI/n$.

X-ray production, target with average atomic number Z ($V \lesssim 5$ MeV):

$$\eta \equiv \text{x-ray power/beam power} = 7 \times 10^{-4} ZV.$$

X-ray dose at 1 meter generated by an e-beam depositing total charge Q coulombs while $V \geq 0.84 V_{\max}$ in material with charge state Z:

$$D = 150 V_{\max}^{2.8} Q Z^{1/2} \text{ rads}.$$